Diploma Programme subject outline—Group 5: mathematics			
School name	Gymnázium, Šrobárova 1, Košice, Slovakia	**School code**	061626
Name of the DP subject *(indicate language)*	DP Mathematics: Applications and Interpretation		
Level *(indicate with X)*	Higher [X] Standard completed in two years [X] Standard completed in one year * [ ]		
Name of the teacher who completed this outline	Katarína Hribová	**Date of IB training**	3 February – 3 March 2021
Date when outline was completed	28 June 2021	**Name of workshop** *(indicate name of subject and workshop category)*	Mathematics: Applications and Interpretation (Cat.1)

* All Diploma Programme courses are designed as two-year learning experiences. However, up to two standard level subjects, excluding languages ab initio and pilot subjects, can be completed in one year, according to conditions established in the *Handbook of procedures for the Diploma Programme*.

1. Course outline

- Use the following table to organize the topics to be taught in the course. If you need to include topics that cover other requirements you have to teach (for example, national syllabus), make sure that you do so in an integrated way, but also differentiate them using italics. Add as many rows as you need.
- This document should not be a day-by-day accounting of each unit. It is an outline showing how you will distribute the topics and the time to ensure that students are prepared to comply with the requirements of the subject.
- This outline should show how you will develop the teaching of the subject. It should reflect the individual nature of the course in your classroom and should not just be a “copy and paste” from the subject guide.
- If you will teach both higher and standard level, make sure that this is clearly identified in your outline.

	Topic/unit (as identified in the IB subject guide) *State the topics/units in the order you are planning to teach them.*	Contents	Allocated time One class is 45 minutes. In one week there are SL: 4 HL:6 classes.	Assessment instruments to be used	Resources *List the main resources to be used, including information technology if applicable.*
Year 1	**Topic 1:** **Number and Algebra**	**SL and HL common content** **SL 1.1** -Operations with numbers in the form $a \times 10^k$ where $1 \leq a < 10$ and k is an integer **SL 1.2** -Arithmetic sequences and series **SL 1.3** -Geometric sequences and series **SL 1.4** -Financial applications of geometric sequences and series: compound interest, annual depreciation **SL 1.5** -Laws of exponents with integer exponents -Introduction to logarithms with base 10 and e **SL 1.6** -Approximation -Upper and lower bounds of rounded numbers -Percentage errors -Estimation **SL 1.7** -Amortization and annuities using technology **SL 1.8** Use technology to solve: -Systems of linear equations in up to 3 variables -Polynomial equations	Recommended teaching hours: SL: 16 hours (21 classes) HL: 29 hours (39 classes)	Assessment revolves around evaluating students´ ability to apply what they have learned in mathematics to real-world contexts. Students will be expected to demonstrate the following: -knowledge and understanding -problem solving -communication and interpretation -technology -reasoning -inquiry approaches Assessment: -will take place during the course -discussions -homework -short tests -end of topic tests -end of chapter tests	Textbooks: • IB Mathematics: Applications and Interpretations SL (OUP) • IB Mathematics: Applications and Interpretations HL (OUP) Online resources: • https://questionbank.ibo.org/ • https://app.kognity.com/ •https://www.thinkib.net/mathapplications • https://resources.ibo.org/home Software/Technology: • GDC • Desmos • GeoGebra • Other software Other resources: • YouTube • Khan Academy • Mathematical Exploration examples • IB Question Bank

	Topic 2: Functions	**HL content** **AHL 1.9** -Laws of logarithms **AHL 1.10** -Simplifying expressions, both numerically and algebraically, involving rational expressions **AHL 1.11** -The sum of infinite geometric sequences **AHL 1.12** -Complex numbers **AHL 1.13** -Modulus-argument polar form: $z = r(\cos\theta + i\sin\theta)$ -Exponential form: $z = re^{i\theta}$ **AHL 1.14** -Matrices **AHL 1.15** -Eigenvalues and eigenvectors - 2×2 matrices **SL and HL common content** **SL 2.1** -Equation of a straight line -Gradients; intercepts **SL 2.2** -Function **SL 2.3** -The graph of a function **SL 2.4** -Key features in graphs -Point of intersection of two curves or lines using technology **SL 2.5** Modelling with the following functions: -Linear models -Quadratic models -Exponential growth and decay models -Direct/inverse variation -Cubic models -Sinusoidal models **SL 2.6** Modelling skills	Recommended teaching hours: SL: 31 hours (41 classes) HL: 42 hours (56 classes)	-end of term progress check tests -short Practice IA Students will be strongly encouraged to self-assess throughout the course. Grade descriptors 7-1: See document DP Grade Descriptors, p.16-17 Also, see the school Assessment policy	

	Topic 3: Geometry and Trigonometry	**HL content** **AHL 2.7** -Composite functions -Inverse functions **AHL 2.8** -Transformation of graphs -Translations -Reflections in the x-axis and in the y-axis -Vertical stretch -Horizontal stretch -Composite transformations **AHL 2.9** Modelling with the following functions: 1.Exponential models to calculate half-life 2.Natural logarithmic models 3.Sinusoidal models 4.Logistic models 5.Piecewise models **AHL 2.10** -Scaling very large or small numbers using logarithms -Linearizing data using logarithms or a power relationship **SL and HL common content** **SL 3.1** -The distance between two points in three dimensional space, and their midpoint -Volume and surface area of three dimensional solids including right-pyramid, right-cone, sphere, hemisphere and combination of these solids -The size of an angle between two interjecting lines or between a line and a plane **SL 3.2** -Use of sine, cosine and tangent ratios to find the sides and angles of right-angled triangles - The sine rule -The cosine rule -Area of a triangle	Recommended teaching hours: SL: 18 hours (24 classes) HL: 46 hours (61 classes)		

		SL 3.3 -Applications of right and non-right angled trigonometry, including Pythagoras´ theorem -Angles of elevation and depression **SL 3.4** -The circle: length of an arc; area of a sector **SL 3.5** -Equations of perpendicular bisectors **SL 3.6** -Voronoi diagrams -Nearest neighbour interpolation -Application of the “toxic waste dump” problem **<u>HL content</u>** **AHL 3.7** -The definition of a radian and conversion between degrees and radians -Using radians to calculate area of sector, length of arc **AHL 3.8** -The definitions of $\cos\theta$ and $\sin\theta$ in terms of the unit circle -The Pythagorean identity -Definition of $\tan\theta$ -Extension of the sine rule to the ambiguous case -Graphical methods of solving trigonometric equations in a finite interval **AHL 3.9** -Geometric transformations of points in two dimensions using matrices -Geometric interpretations of the determinant of a transformation matrix **AHL 3.10** -Vectors and scalars **AHL 3.11** -Vector equation of a line in 2D and 3D **AHL 3.12** -Vector applications to kinematics -Modelling linear motion			

		AHL 3.13 -Scalar product of two vectors -The angle between two vectors; the acute angle between two lines -Vector product of two vectors -Geometric interpretation of $\|v \times w\|$ **AHL 3.14** -Graph theory -Simple graphs; complete graphs; weigted graphs -Directed graphs -Subgraphs; trees **AHL 3.15** -Adjacency matrices -Walks -Weighted adjacency tables -Construction of the transition matrix for a strongly-connected, undirected or directed graph **AHL 3.16** -Tree and cycle algorithms with undirected graphs -Walks, trails, paths, circuits, cycles -Eulerian trails and circuits -Hamiltonian paths and cycles -Minimum spanning tree (MST) graph algorithms -Kruskal´s and Prim´s algorithms -Chinese postman problem and algorithm -Travelling salesman probem to determine the Hamiltonian cycle of least weight in a weighted complete graph -Nearest neighbour algorithm -Deleted vertex algorithm			
	Introduction to Mathematical Exploration **Revision**	**<u>SL and HL</u>** -Discuss what the Internal Assessment is, the criteria for grading, academic honesty, samples, possible topics -Practice AI -Past paper questions practice	Recommended teaching hours: SL/HL: 10 hours (SL/HL: 13 classes) (SL: 8 classes) (HL: 4 classes)		

Year 1 & Year 2	**Topic 4:** **Statistics and Probability**	**<u>SL and HL common content</u>** **SL 4.1** -Concepts of population, sample, random sample, discrete and contiuous data -Reliability of data source and bias in sampling -Interpretation of outliers -Sampling techniques and their effectiveness **SL 4.2** -Presentation of data (discrete and continuous):frequency distributions (table) -Histograms -Cumulative frequency graphs -Box and whisker diagrams **SL 4.3** -Measures of central tendency -Estimation of mean from grouped data -Modal class -Measures of dispersion -Effect of constant changes to the original data -Quartiles of discrete data **SL 4.4** -Linear correlation of bivariate data -Pearson´s product-moment correlation coefficient r -Scatter diagrams; lines of best fit -Equation of the regression line of y on x **SL 4.5** -Concepts of trial, outcome, equally likely outcomes, relative frequency, sample space (U) and event -The probability of an event A -The complementary events A, A' -Expected numbers of occurences **SL 4.6** -Venn diagrams, tree diagrams, sample space diagrams and tables of outcomes -Combined events -Mutually exclusive events -Conditional probability -Independent events	Recommended teaching hours: SL: 16 + 20 hours (21 + 27 classes) HL: 14 + 38 hours (19 + 50 classes)	Assessment revolves around evaluating students´ ability to apply what they have learned in mathematics to real-world contexts. Students will be expected to demonstrate the following: -knowledge and understanding -problem solving -communication and interpretation -technology -reasoning -inquiry approaches Assessment: -will take place during the course -discussions -homework -short tests -end of topic tests -end of chapter tests - end of term progress check tests -Final IA (tbc) -Final Exam (May) Students will be strongly encouraged to self-assess throughout the course.	Textbooks: • IB Mathematics: Applications and Interpretations SL (OUP) • IB Mathematics: Applications and Interpretations HL (OUP) Online resources: • https://questionbank.ibo.org/ • https://app.kognity.com/ •https://www.thinkib.net/mathapplications • https://resources.ibo.org/home Software/Technology: • GDC • Desmos • GeoGebra • Other software Other resources: • YouTube • Khan Academy • Mathematical Exploration examples • IB Question Bank Other resources: • YouTube • Khan Academy • Mathematical Exploration examples • IB Question Bank

		SL 4.7 -Discrete random variables and their probability distributions -Expected value (mean), $E(X)$ for discrete data **SL 4.8** -Binomial distribution **SL 4.9** -The normal distribution and curve -Normal probability representations -Inverse normal calculations **SL 4.10** -Spearman´s rank correlation coefficient -Awareness of the apropriateness and limitations of Pearson´s product moment correlation coefficient and Spearman´s rank correlation coefficient, and the effect of outliers on each **SL 4.11** -Formulation of null and alternative hypotheses -Significant levels -p-values -Expected and observed frequences -The χ^2 test for independence: contingency tables, degree of freedom, critial value -The χ^2 goodness of fit test -The t-test -Use of the p-value to compare the means of two populations -Using one-tailed and two-tailed tests **HL content** **AHL 4.12** -Data collection methods: surveys and questionnaires -Selecting relevant variables from many variables -Chosing relevant and appropriate data to analyse -Categorising numerical data in a χ^2 table -Reliability and validity + tests		IA Assessment criteria: A – Presentation B – Mathematical communication C – Personal Engagement D – Reflection E – Use of Mathematics Final Exam and Assessment outline (SL): -External Paper 1, 90 min., 40% Paper 2, 90 min., 40% -Internal Mathematical Exploration, 20% Final Exam and Assessment outline (HL): -External Paper 1, 120 min., 30% Paper 2, 120 min., 30% Paper 3, 60 min., 20% -Internal Mathematical Exploration, 20% Grade descriptors 7-1: See document DP Grade Descriptors, p.16-17 Also, see the school Assessment policy	

		AHL 4.13 -Non-linear regression -Evaluating of least squares regression curves using technology -Sum of square residuals (SS_{res}) -The coefficient of determination (R^2) **AHL 4.14** -Linear transformation of a single random variable -Expected value of linear combinations of n random variables -Variance of linear combinations of n independent random variables - $\tilde{x}$ as an unbiased estimate of μ - s_{n-1}^2 as an unbiased estimate of σ^2 **AHL 4.15** -A linear combination of n independent normal random variables is normally distributed -Central limit theorem **AHL 4.16** -Confidence intervals for the mean of a normal population **AHL 4.17** -Poisson distribution, its mean and variance -Sum of two independent Poisson distributions **AHL 4.18** -Critical values and critical regions -Test for population mean for normal distribution -Test for proportion using binomial distribution -Test for population mean using Poisson distribution -Use of technology to test the hypothesis that the population product moment correlation coefficient is 0 for bivariate normal distributions -Type I and II errors including calculations of their probabilities **AHL 4.19** -Transition matrices -Regular Markov chains -Initial state probability matrices -Steady state and long-term probabilities			

Year 2	**Topics 5: Calculus**	**<u>SL and HL common content</u>** **SL 5.1** -Introduction to the concept of a limit -Derivative interpreted as gradient function and as rate of change **SL 5.2** -Increasing and decreasing functions -Graphical interpretation of $f'(x) > 0, f'(x) = 0, f'(x) < 0$ **SL 5.3** -Derivative of $f(x) = ax^n$ is $f'(x) = anx^{n-1}, n \in \mathbb{Z}$ -The derivative of functions in the form $f(x) = ax^n + bx^{n-1} + \cdots$ where all exponents are integers **SL 5.4** -Tangents and normals **SL 5.5** -Introduction to the integration as anti-differentiation of functions of the form $f(x) = ax^n + bx^{n-1} + \cdots$, where $n \in \mathbb{Z}, n \neq -1$ -Anti-differentiation with a bundary condition to determine the constant term -Definite integrals using technology -Area of a region enclosed by a curve $y = f(x)$ and the x-axis, where $f(x) > 0$ **SL 5.6** -Values of x where the gradient of a curve is zero -Solution of $f'(x) = 0$ -Local maximum and minimum points **SL 5.7** -Optimisation problems in context **SL 5.8** -Trapezoidal rule	Recommended teaching hours: SL: 19 hours (26 classes) HL: 41 hours (55 classes)		

		HL content **AHL 5.9** -The derivatives of $\sin x$, $\cos x$, $\tan x$, e^x, $\ln x$, x^n, where $n \in \mathbb{Q}$ -The chain rule, product and quotient rules -Related rates of change **AHL 5.10** -The second derivative -Use the second derivative test to distinguish between a maximum and a minimum point **AHL 5.11** -Definite and indefinite integration of x^n where $n \in \mathbb{Q}$, including $n = -1$, $\sin x$, $\cos x$, $\frac{1}{cos^2 x}$ and e^x -Integration by inspection, or substitution of the form $\int f\big(g(x)\big)g'(x)dx$ **AHL 5.12** -Area of the region enclosed by a curve and the x or the y-axes ina given interval -Volumes of revolution about the x-axis or y-axis **AHL 5.13** -Kinematic problems involving displacement s, velocity v and acceleration a **AHL 5.14** -Setting up a model/differential equation from context -Solving by separation of variables **AHL 5.15** -Slope fields and their diagrams **AHL 5.16** -Euler´s method for finding the approximate solution to first order differential equations -Numerical solution of $\frac{dy}{dx} = f(x, y)$ -Numerical solutions of the coupled system $\frac{dx}{dt} = f_1(x, y, t)$ and $\frac{dy}{dt} = f_2(x, y, t)$			

		AHL 5.17 -Phase portrait for the solutions of coupled differential equations of the form: $\frac{dx}{dt} = ax + by;\ \frac{dy}{dt} = cx + dy$ -Qualitative analysis of future paths for distinct, real, complex and imaginary eigenvalues -Sketching trajectories and using phase portraits to identify key features such as equilibrium points, stable populations and saddle points. **AHL 5.18** -Solutions of $\frac{d^2x}{dt^2} = f(x, \frac{dx}{dt}, t)$ by Euler´s method			
	Final Mathematical Exploration **Exam Revision and Preparation**	**<u>SL and HL</u>** -Completing the final Mathematical Exploration IA -In preparation for the final exams, students will revise using past paper questions, past papers and past assignments	Recommended teaching hours: SL/HL: 20 hours (SL/HL: 27 classes) (SL: 12 classes) (HL: 6 classes)		

2. IB internal assessment requirement to be completed during the course

Briefly explain how and when you will work on it. Include the date when you will first introduce the internal assessment requirement to your students, the different stages and when the internal assessment requirement will be due.

The internal assessment (IA) is an individual exploration. This is a piece of written work that involves investigating an area of mathematics.
- Students willl be introduced to Mathematical Exploration during the second term of Year 1
- Students will do a short practice Mathematical Exploration Internal Assessment in Year 1 (6-8 pages)
- Students willl continue to work on their Mathematical Exploration during Year 2
- Students are expected to complete their Final Mathematical Exploration Internal Assessment in Year 2 (deadline : see school calendar)

3. Links to TOK

You are expected to explore links between the topics of your subject and TOK. As an example of how you would do this, choose one topic from your course outline that would allow your students to make links with TOK. Describe how you would plan the lesson.

Topic	Link with TOK (including description of lesson plan)
Topic 1: Number and Algebra **SL 1.2** Arithmetic sequences and series	TOK : Is all **knowledge** concerned with identification and use of patterns ? Consider Fibonacci numbers and connections with the Golden ratio. Aim : Explore knowledge via the perspectives + methods and tools elements in mathematics from the TOK curriculum, considering Fibonacci numbers and connections with the Golden ratio. (Note : The lesson follows cooperative planning with the ToK teacher.) Activity : Students will look at how mathematics often appears in nature. They will start by recalling Fibonacci numbers and their sequence. Then students will be presented with some pictures to explore the Golden ratio (pair/group work). They will present their findings to the rest of the class. Connections will be drawn between the Fibonacci numbers and the Golden ratio. Students will find their own examples and think of other possible applications.

4. Approaches to learning

Every IB course should contribute to the development of students' approaches to learning skills. As an example of how you would do this, choose one topic from your outline that would allow your students to specifically develop one or more of these skill categories (thinking, communication, social, self-management or research).

Topic	Contribution to the development of students' approaches to learning skills (including one or more skill category)
Topic 4: Statistics and Probability **AHL 4.12** Data collection methods: surveys and questionnaires **SL 4.1 – SL 4.4 or other**	Students will choose a research topic of their interest, design a questionnaire and conduct their own survey (among their peers, family members, members of the public,…). After collecting their data, they will analyse it, present it graphically and draw conclusions. They will make a short presentation about their research to their classmates and answer their questions.

5. International mindedness

Every IB course should contribute to the development of international-mindedness in students. As an example of how you would do this, choose one topic from your outline that would allow your students to analyse it from different cultural perspectives. Briefly explain the reason for your choice and what resources you will use to achieve this goal.

Topic	Contribution to the development of international mindedness (including resources you will use)
Topic 5: Calculus **SL 5.1** Introduction to the concept of a limit	International-mindedness : Attempts by Indian mathematicians (500-1000 CE) to explain division by zero. Students will explore the history and origin of zero, operations with zero and the connection with the concept of a limit. e.g. You Tube video : The story of zero – getting something from nothing ; The Royal Institution https://aeon.co/videos/a-history-of-nothing-how-zero-went-from-nil-to-something Other resources will include various websites and books about the history of mathematics. Reason for choice : Zero is a fascinating concept and nowadays we take it for granted. I think it is important for students to see how the concept of zero developed and how it was perceived and used in different cultures.

5. Development of the IB learner profile

Through the course it is also expected that students will develop the attributes of the IB learner profile. As an example of how you would do this, choose one topic from your course outline and explain how the contents and related skills would pursue the development of any attribute(s) of the IB learner profile that you will identify.

Topic	Contribution to the development of the attribute(s) of the IB learner profile
Topic 3 : Geometry and Trigonometry **SL 3.2** Sine, cosine, tangent ratios ; Sine rule, Cosine rule, Area of triange	Inquirers – Students will look at different types of triangles and ask curious questions about their connection with different trigonometry ratios and rules. Knowledgeable – Students use their understanding of the concepts and explore the application of their knowledge in contexts such as triangulation, map-making and other subjects such as physics. Thinkers – Students will work on complex problems applying their knowledge of different trigonometry ratios and rules Communicators – Students will be expected to communicate their ideas individually using correct mathematical writing but also with confidence and collaboratively as a part of a group when working in pairs/groups. Principled – In all areas of this topic, students are expected to sketch well-labelled diagrams to support their solutions and to work out the answers systematically : write down the formula, substitute, calculate, give answer including units Open-minded – Students will learn about the occurrence of Pythagoras´theorem in early Chinese and Indian manuscripts ad the earliest references to trigonomentry in Indian mathematics. Reflective – Students will reflect upon their work, identify misconceptions and seek ways for improvement.

6. Resources

Describe the resources that you and your student will have to support the subject. Indicate whether they are sufficient in terms of quality, quantity and variety. Briefly describe what plans are in place if changes are needed.

I have come across the following resource suggestions during the IB workshop. Some are free and some require purchase / subscription. Some textbooks will be used for reference and as teacher planning material, some will be used by students as the study material. The school is supportive in purchasing the resources needed and sufficient funds have been allocated.

Textbooks :

- IB Mathematics: Applications and Interpretations SL (OUP)
- IB Mathematics: Applications and Interpretations HL (OUP)
- Mathematics: Applications and Interpretations SL (Pearson)
- Mathematics: Applications and Interpretations HL (Pearson)
- Mathematics for the IB Diploma: Applications and Interpretation SL (Hodder)
- Mathematics for the IB Diploma: Applications and Interpretation HL (Hodder)
- Mathematics Core Topics SL1 (Haese)
- Mathematics Applications and Interpretations SL2 (Haese)
- Mathematics Core Topics HL1 (Haese)
- Mathematics Applications and Interpretations HL2 (Haese)

Online resources:

- https://questionbank.ibo.org/
- https://app.kognity.com/
- https://www.thinkib.net/mathapplications
- https://resources.ibo.org/home

Software/Technology:

- GDC
- Desmos
- GeoGebra
- Other software (Word, Excel, Powerpoint, apps, …)

Other resources:

- YouTube
- Khan Academy
- Mathematical Exploration examples
- IB Question Bank